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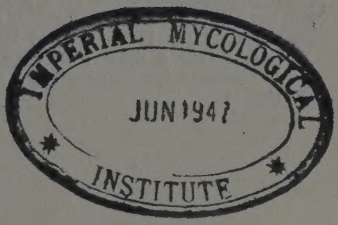

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MATHEMATICS

THE THEORY OF GEOMETRIC OBJECTS AND THE THEORY OF FINITE AND INFINITE CONTINUOUS GROUPS OF TRANSFORMATIONS

By V. WAGNER

(Communicated by A. N. Kolmogoroff, Member of the Academy, 17. XII. 1944)

The notion of geometric objects is one of the most important in the modern differential geometry. Nevertheless there are comparatively few papers dealing with the general theory of geometric objects. The first exact definition of geometric object was given by Schouten and van Dantzig (1) in 1935. The first attempt to establish the general theory of geometric objects was made by Wundheiler (2). Further the general theory of geometric objects was developed by Schouten and Haantjes (3). But none of these writers has paid attention to the fact that in an implicit form geometric objects of general kind were considered already in the last century in connexion with the theory of infinite continuous groups of transformations (4). The method of Medolaghi for obtaining all the continuous groups of transformations (finite and infinite) gives the method for obtaining all types of geometric objects of the class v (5) which are the sole geometric objects really employed in differential geometry. The purpose of the present note is to establish this connexion between the general theory of geometric objects and the theory of continuous groups of transformations from the modern point of view.

Consider the n-dimensional geometric space X_n with point coordinates ζ^α whose allowable transformations determine the group

$$'\xi^\alpha = f^\alpha(\xi^\lambda) \tag{1}$$

of all the analytical point transformations in the n-dimensional arithmetic space of the variables ξ^α. Let $\Omega^i\,(i=1,\ldots,N)$ be a geometric object of the class v in X_n, whose transformation of components is given by the equations

$$'\Omega^i = F^i\left(\Omega_j, \xi^\alpha, f^\alpha(\xi^\lambda), f^\alpha_{\beta_1\ldots\beta_s}(\xi^\lambda)\right)^{(s=1\ldots v)}_{(\beta_1\leqslant\beta_2\leqslant\ldots\leqslant\beta_s)} \tag{2}$$

Geometric objects with the same law of transformation of the components are said to be of the same type. Considering Ω^i as independent variables, we shall find that the equations (1) and (2) together define an infinite group of point transformations in the arithmetic space of the $n+N$ variables ξ^α, Ω^i. This group is an extension of the group (1). Inversely, such extension of the group (1) introducing N new variables determines a certain type of geometric objects with N components.

The group of transformations (1), (2) is said to be the total group of the considered type of geometric objects Ω^i. Two types of geometric objects are said to be similar if their total groups are similar. The total group is imprimitive; the equations $\xi^\alpha = \text{const}$ determine its surface of imprimitivity.

Consider the subgroup of total group which preserves invariant a certain surface of imprimitivity $\xi^\alpha = \xi^\alpha_0$. This subgroup induces on the surface a finite group

$$'\Omega^i = F^i(\Omega^j, \xi^\alpha_0, \xi^\alpha_0, f^\alpha_{\beta_1 \ldots \beta_s}(\xi^\lambda_0)) \tag{3}$$

called the local group at the point ξ^α_0 of the type of geometric objects under consideration.

A geometric object is said to be a differential geometric object if the functions F^i in (2) do not involve ξ^α and $f^\alpha(\xi^\lambda)$ explicitly, *i. e.* the variations (2) are of the form

$$'\Omega^i = F\ (\Omega\ , f^\alpha_{\beta_1 \ldots \beta_s}(\xi^\lambda)) \tag{4}$$

All the geometric objects employed in differential geometry are differential geometric objects except the point ξ^α which is a geometric object of class 0 belonging to the type of geometric objects defined by the equations

$$'\Omega^\alpha = \Omega^\alpha + f^\alpha(\xi^\lambda) - \xi^\alpha \tag{5}$$

It can be proved that each type of geometric object is similar to a type of differential geometric object. Further we confine ourselves to the study of different geometric objects.

Let us consider the general case where the local group of the type of differential objects under consideration may be intransitive, and let L be the number of dimensions of its surfaces of intransitivity. Then the type of geometric objects is similar to a type of the following form

$$\left.\begin{array}{l} '\Omega^a = F^a(\Omega^b, \Omega^h, f^\alpha_{\beta_1 \ldots \beta_s}(\xi^\lambda)) \\ \\ '\Omega^h = \Omega^h; \quad '\Omega^p = \Omega^p \end{array}\right\} \begin{array}{l} a,\ b = 1, \ldots, L \\ h = L+1, \ldots, M \\ p = M+1, \ldots, N \end{array} \tag{6}$$

where Ω^h are essentially involved in the first set of the equations (6), *i. e.* their number cannot be diminished by a transformation of the variables. Consider the type of differential geometric objects with $N_{nv} = n\left[\frac{(n+1)\ldots(n+v)}{1\ldots v} - 1\right]$ components whose components transform in the same way as the partial derivatives up to the v-th order of n scalar functions. We denote the components of this geometric object by $a^\alpha_{\beta_1}, \ldots, a^\alpha_{\beta_1 \ldots \beta_v}$. The local group of this type of geometric objects is a simply transitive group which will be denoted symbolically by $\mathfrak{A}_{N_{nv}}$.

Besides we consider a type of differential geometric objects also with N_{nv} components whose components transform in the same way as the partial derivatives up to the v-th order of point coordinates ξ^α considered as functions of n invariant parameters. We denote the components of this geometric objects by $b^\alpha_{\beta_1}, \ldots, b^\alpha_{\beta_1, \ldots \beta_v}$. The local group of this type of geometric objects is a simply transitive group which will be denoted symbolically by $\mathfrak{B}_{N_{nv}}$. It can be easily seen that the groups $\mathfrak{A}_{N_{nv}}$ and $\mathfrak{B}_{N_{nv}}$ are reciprocal.

The equations

$$F^a(\overset{o}{\Omega}{}^b, \overset{o}{\Omega}{}^h, b^\alpha_{\beta_1 \ldots \beta_s}) = \Omega^a \tag{7}$$

where $\overset{o}{\Omega}{}^b, \overset{o}{\Omega}{}^h$ are fixed and Ω^a are considered as variable parameters, determine a system of surfaces of imprimitivity for the group $\mathfrak{B}_{N_{nv}}$. Replacing $b^\alpha_{\beta_1 \ldots \beta_s}$ by $a^\alpha_{\beta_1 \ldots \beta_s}$ we obtain the equations determining the complete system of invariants for a subgroup of the group $\mathfrak{A}_{N_{nv}}$. It can be easily deduced from this that the problem of finding all types of geometric objects in X_n is equivalent to the finding of all systems of imprimitivity for the simply transitive group $\mathfrak{B}_{N_{nv}}$, or, what is the same, is equivalent to the finding of invariants of all subgroups of the simply transitive group $\mathfrak{A}_{N_{nv}}$.

Consider a point transformation $*\xi^\alpha = \varphi^\alpha(\xi^\lambda)$ in X_n. Under this transformation every geometric object $\Omega^i = \Omega^i(\xi^\alpha)$ is transformed into the geometric

object ${}^*\Omega^i = {}^*\Omega^i(\xi^\alpha)$ defined by the equations

$$ {}^*\Omega^i(\varphi^\alpha(\xi^\lambda)) = F^i(\Omega^j(\xi^\lambda), \varphi^\alpha_{\beta_1\ldots\beta_s}(\xi^\lambda)) \tag{8} $$

If ${}^*\Omega^i$ coincides with Ω^i, *i. e.*

$$ \Omega^i(\varphi^\alpha(\xi^\lambda)) = F^i(\Omega^j(\xi^\lambda), \varphi^\alpha_{\beta_1\ldots\beta_s}(\xi^\lambda)) \tag{9} $$

the geometric object Ω^i is said to be invariant under the corresponding point transformation in X_n. The totality of transformations which preserve invariant the geometric object Ω^i determines Lie's continuous group of transformations which can be finite or infinite (in particular cases it can reduce to the identical transformation). This group is said to be the group of invariancy of the geometric object Ω^i. The equations (9) are its defining equations. Being solved for $\Omega^i(\xi^\lambda)$ they can be represented thus

$$ \Phi^i(\Omega^j(\varphi^\alpha(\xi^\lambda)), \varphi^\alpha_{\beta_1\ldots\beta_s}(\xi^\lambda)) = \Omega^i(\xi^\lambda) \tag{10} $$

Medolaghi has shown (6) that the defining equations of any group can be represented in the form (10). From the modern point of view this theorem of Medolaghi means that each continuous group of Lie (finite or infinite) is the group of invariancy of a certain differential geometric object.

Consider one-parameter family of point transformations in X_n

$$ {}^*\xi^\alpha = \varphi^\alpha(\xi^\lambda, t) \tag{11} $$

containing the identical transformation for $t=0$. The transformations (11) determine the infinitesimal transformation corresponding to the vector $V^\alpha(\xi^\lambda) = [\partial\varphi^\alpha/dt]_{t=0}$. Transforming a geometric object Ω^i by means of the point transformation (11) we obtain a one-parameter set of geometric objects ${}^*\Omega^i(\xi^\lambda, t)$. The object $\overset{L}{D}\Omega^i$ where

$$ \overset{L}{D}\Omega^i = -\left(\frac{\partial {}^*\Omega^i}{\partial t}\right)_{t=0} = V^\omega \partial_\omega \Omega^i - \sum_{s=1}^{v} F^i_{0\,\varphi^\alpha_{\beta_1\ldots\beta_s}} \partial_{\beta_1}\cdots\partial_{\beta_s} V^\alpha. $$

$$ F^i_{0\,\varphi^\alpha_{\beta_1\ldots\beta_s}} = F^i_{\varphi^\alpha_{\beta_1\ldots\beta_s}}(\Omega^j, \delta^\alpha_\beta, 0 \ldots 0) \tag{12} $$

is said to be the Lie derivative (7) of the geometric object Ω^i with respect to the infinitesimal transformation V^α. It follows from this definition that if the transformations (11) belong to the group of invariancy of the geometric object Ω^i; its Lie derivative vanishes. Thus, the equations of definition for the infinitesimal transformations of the group of invariancy of Ω^i can be written:

$$ \overset{L}{D}\Omega^i = 0 \tag{13} $$

Before Medolaghi Engel (8) has found a general canonic form of all the equations of definition for the infinitesimal transformations of continuous groups. From the modern point of view the equations of definition for the infinitesimal transformations represented in the Engel form express the vanishing of the Lie derivative of the corresponding differential geometrical object with respect to all the infinitesimal transformations contained in the group.

Saratov University.

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MATHÉMATIQUES

SUR UNE FORMULE SOMMATOIRE LIÉE À LA TRANSFORMATION DE HANKEL

Par M. OLEVSKY

(Présenté par S. L. Soboleff, de l'Académie, le 13. III. 1944)

1. Soit

$$F(x) = \int_{-\infty}^{\infty} f(y)\, e^{2\pi i x y}\, dy \tag{1}$$

Alors on sait que si $f(x)$ est assujettie à certaines conditions restrictives (§ 2), $\sum_{n=-\infty}^{\infty} f(n)$ est invariante par rapport à la transformation (1), c'est-à-dire

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} F(n) \tag{2}$$

Le but de la présente Note est de donner une formule (4) analogue pour la transformation de Hankel

$$\Phi(x) = 2\pi \int_{0}^{\infty} \varphi(y)\, y J_p(2\pi x y)\, dy \tag{3}$$

pour les valeurs de $p > -1$, p étant entier ou la moitié d'un entier, et d'indiquer certaines conséquences de cette formule.

2. Théorème. *Supposons que $\varphi(t)$ vérifie les conditions suivantes:*

a) *$t^{-p}\varphi(t)$ est bornée, intégrable dans $(0, \infty)$ et continue pour les valeurs de t représentables par la forme définie positive $A(x, x) \equiv \sum_{i,j=1}^{k} a_{ij} x_i x_j$, $k = 2p+2$, $p > -1$ dont les coefficients a_{ij} entiers sont donnés et les x^i sont entiers;*

b) *pour $t \to \infty$ on a $\varphi(t) = O\left(t^{-\frac{1}{2}k-1-\varepsilon}\right)$, $\varepsilon > 0$;*

c) *pour $n \to \infty$ on a $\Phi(n) \equiv 2\pi \int_{0}^{\infty} \varphi(t)\, t J_p(2\pi n t)\, dt = O\left(n^{-\frac{1}{2}k-1-\eta}\right)$, $\eta > 0$*

Désignons par $B(x,x)$ la forme $\sum_{i,j=1}^{k} A_{ij} x_i x_j$ où $\frac{A_{ij}}{D(A)}$ est l'élément de la matrice A^{-1} inverse à $A = \|a_{ij}\|$, $D(A)$ est le déterminant de la matrice A, et par $r_A(n)$ le nombre de solutions entières de l'équation $\sum_{i,j=1}^{k} a_{ij} x_i x_j = n$.

*Alors on a l'égalité**

$$\sum_{n=0}^{\infty} r_A(n)\,\frac{\varphi(\sqrt{n})}{(\sqrt{n})^p} = \frac{1}{\sqrt{D(A)}} \sum_{n=0}^{\infty} r_B(n)\,\frac{\Phi\left(\sqrt{\frac{n}{D(A)}}\right)}{\left(\sqrt{\frac{n}{D(A)}}\right)^p} \tag{4}$$

qui présente, en abrégé, l'égalité suivante:

$$\lim_{n\to+0} \frac{\varphi(\sqrt{n})}{(\sqrt{n})^p} + \sum_{n=1}^{\infty} r_A(n)\,\frac{\varphi(\sqrt{n})}{(\sqrt{n})^p} =$$

$$= \frac{1}{\sqrt{D(A)}} \left[\frac{2\pi^{p+1}}{\Gamma(p+1)} \int_0^{\infty} \varphi(t)\, t^{p+1}\, dt + \sum_{n=1}^{\infty} r_B(n)\,\frac{\Phi\left(\sqrt{\frac{n}{D(A)}}\right)}{\left(\sqrt{\frac{n}{D(A)}}\right)^p} \right] \tag{4'}$$

où les deux séries sont absolument convergentes.

Démonstration. Nous partons de la formule sommatoire généralisée de Poisson

$$\sum_n f(n_1, \ldots, n_k) = \sum_n F(n_1, \ldots, n_k) \tag{5}$$

où

$$F(n_1, \ldots, n_k) = \int_{-\infty}^{\infty} \cdots \int f(x_1, \ldots, x_k)\, e^{2\pi i (n,x)}\, dx_1 \ldots dx_k, \quad k \geqslant 1^{**}, \tag{6}$$

Pour qu'elle soit valable il suffit que les conditions suivantes soient vérifiées (2):

α) $f(x_1, \ldots, x_k)$ est bornée, intégrable dans $-\infty < x_i < \infty$ $(i = 1, 2, \ldots, k)$ et continue pour les valeurs entières de $x_1, \ldots, x_k$;

β) la série $\sum_n f(x_1 + n_1, \ldots, x_k + n_k)$ converge uniformément dans $-\frac{1}{2} \leqslant x_i \leqslant \frac{1}{2}$ $(i = 1, 2, \ldots, k)$;

γ) la série $\sum_n F(n_1, \ldots, n_k)$ converge.

Dans les conditions α) et β) la série de Fourier de la fonction

$$\psi(x_1, \ldots, x_k) \equiv \sum_n f(x_1 + n_1, \ldots, x_k + n_k)$$

est (aux points de continuité de ψ) sommable vers cette fonction par la méthode C 1:

$$\psi(x_1, \ldots, x_k) = \sum_m e^{-2\pi i (m,x)} \int_{-\frac{1}{2}}^{\frac{1}{2}} \cdots \int \psi(y_1, \ldots, y_k)\, e^{2\pi i (m,y)}\, dy_1 \ldots dy_k =$$

$$= \sum_m e^{-2\pi i (m,x)} \int_{-\infty}^{\infty} \cdots \int f(y_1, \ldots, y_k)\, e^{2\pi i (m,y)}\, dy_1 \ldots dy_k$$

---

* Un cas particulier de (4) pour $p = 0$, $a_{ij} = \delta_{ij}$ et la fonction φ nulle en dehors de l'intervalle fini $0 \leqslant \alpha \leqslant t \leqslant \beta$ se trouve dans le livre de Landau (1). La formule sommatoire de Poisson (pour cos-transformation de Fourier) est continue dans (4) pour $k = 1$ et $a_{11} = 1$.

** $(n, x) \equiv n_1 x_1 + \ldots + n_k x_k$; le signe $\sum_n$ désigne la sommation étendue à tous les $n_1, \ldots, n_k$ de $-\infty$ à $+\infty$.

Il en résulte que pour $x_1 = \ldots = x_k = 0$ on a (5) en vertu de γ) Cette démonstration de la formule (5) montre que la condition γ) peut être omise si l'on fait la sommation de la série $\sum_n F(n_1, \ldots, n_k)$ dans (5) par la méthode C 1*.

Pour démontrer (4) posons dans (5)

$$f(x_1, \ldots, x_k) = \varphi_1(r) \tag{7}$$

où $r = \sqrt{\sum_{i,j=1}^{k} a_{ij} x_i x_j}$. Il est facile à voir que dans le second membre de (6) l'intégrale multiple peut être réduite dans ce cas à une intégrale ordinaire. En effet, en introduisant au lieu des x des variables y au moyen de la transformation linéaire $x = Ly$ qui transforme $A(x, x)$ en une somme de carrés, nous avons

$$F(n_1, \ldots, n_k) = \int_{-\infty}^{\infty} \ldots \int \varphi_1(\sqrt{(y, y)})\, e^{2\pi i (n, Ly)} D(L)\, dy_1 \ldots dy_k$$

Mais $(n, Ly) = (N, y)$ où $N = L'n$, et d'ailleurs $(N, N) = (A^{-1}n, n)$ en vertu de ce que $L'AL = E$.

En introduisant maintenant au lieu de $y_1, \ldots, y^k$ les coordonnées polaires généralisées $r, \theta_1, \ldots \theta_{k-1}$, nous avons

$$\int_{-\infty}^{\infty} \ldots \int f(x_1, \ldots, x_k)\, e^{2\pi i (n, x)}\, dx_1, \ldots, dx_k =$$

$$= \frac{1}{\sqrt{D(A)}} \int_0^{\infty} \varphi_1(r)\, r^{k-1} \left[\int_{\Omega_k} \ldots \int e^{2\pi i (N, y)}\, d\omega_k \right] dr$$

L'intégrale intérieure est étendue à la surface Ω_k de la sphère à k dimensions de rayon 1 et peut être calculée directement, car, comme elle est invariante par rapport aux transformations orthogonales, elle se réduit à

$$\frac{2\pi^{\frac{k-1}{2}}}{\Gamma\left(\frac{k-1}{2}\right)} \int_0^{\pi} e^{2\pi i |N| r \cos\theta_{k-1}} \sin^{k-2}\theta_{k-1}\, d\theta_{k-1}, \quad |N| = \sqrt{(A^{-1}n, n)}$$

donc nous avons

$$\int_{-\infty}^{\infty} \ldots \int f(x_1, \ldots, x_k)\, e^{2\pi i (n, x)}\, dx_1 \ldots, dx_k =$$

$$= \frac{2\pi}{\sqrt{D(A)}} |N|^{-\frac{1}{2}(k-2)} \int_0^{\infty} r^{\frac{1}{2}k} J_{k-2}(2\pi |N| r) \varphi_1(r)\, dr \tag{8}$$

Ainsi nous voyons que si la condition (7) est vérifiée, $F(n_1, \ldots, n_k)$ est une fonction de $|N|$, $\frac{1}{\sqrt{D(A)}} \Phi_1(|N|)$, et que $\Phi(|N|)$ est la transformation (3) de

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* Si $f(n_1 - 0, \ldots, n_k - 0)$ et $f(n_1 + 0, \ldots, n_k + 0)$ existent, on peut dans la condition α) remplacer la continuité par l'hypothèse que la fonction est composée d'un nombre fini de fonctions continues, pourvu que l'on remplace $f(n_1, \ldots, n_k)$ dans le premier membre de (5) par la valeur $\frac{1}{2}[f(n_1 - 0, \ldots, n_k - 0) + f(n_1 + 0, \ldots, n_k + 0)]$.

Hankel pour la fonction $\varphi(t)$ où

$$\varphi(t) = t^p \varphi_1(t), \quad \Phi(t) = t^p \Phi_1(t) \tag{9}$$

En tenant compte de ce que la série $\sum_n (An, n)^{-\mu}$ converge pour $2\mu > k$ puisque $(An, n) \geqslant \lambda_1 (n, n)$, où $\lambda_1 > 0$ est la plus petite valeur caractéristique de la matrice A, nous trouvons facilement que les conditions α), β) et γ) sont vérifiées en vertu des hypothèses a) et b) sur la fonction φ, et ainsi l'égalité (5) donne

$$\sum_n \varphi_1\left(\sqrt{\sum_{i,j=1}^{k} a_{ij} n_i n_j}\right) = \frac{1}{\sqrt{D(A)}} \sum_n \Phi_1\left(\sqrt{\frac{1}{D(A)} \sum_{i,j=1}^{k} A_{ij} n_i n_j}\right)^* \tag{10}$$

En vertu de (9) et b) la série du premier membre converge absolument, donc elle peut être écrite dans la forme $\sum_{n=0}^{\infty} r_A(n) \frac{\varphi(\sqrt{n})}{(\sqrt{n})^p}$ (la $\lim_{n \to +\infty} \varphi(n)/n^p$ existe en vertu de la condition a)), tandis qu'en vertu de c) la série du second membre converge aussi absolument et est égale à

$$\sum_{n=0}^{\infty} r_B(n) \left(\sqrt{\frac{D(A)}{n}}\right)^p \Phi\left(\sqrt{\frac{n}{D(A)}}\right) \quad \text{c. q. f. d.}$$

Corollaire. Pour $a_{ij} = \delta_{ij}$ l'expression $\sum_{n=0}^{\infty} r_k(n) \frac{\varphi(\sqrt{n})}{(\sqrt{n})^p}$, où $r_k(n)$ est le nombre de solutions entières de l'équation $x_1^2 + \ldots + x_k^2 = n$**, est un invariant par rapport à la transformation (3) de Hankel.

Remarque. Les conditions du théorème peuvent être affaiblies si l'on effectue la sommation des séries en (4) par une méthode de Cesaro d'ordre assez élevé (ou bien par la méthode de Riesz qui lui est équivalente et qu'il est plus commode d'appliquer dans la démonstration correspondante).

3. La formule (4) quand on y considère des expressions spéciales en qualité de $\varphi(x)$ est une source d'identités curieuses. Ainsi, par exemple, en posant $\varphi(x) = e^{-ax^2} J_p(bx)$, $a > 0$, on obtient une généralisation de la θ-relation (la θ-relation classique est obtenue pour $p = -\frac{1}{2}$ et $a_{11} = 1$).

Posons $\varphi(x) = \begin{cases} x^p, & 0 \leqslant x < \sqrt{\omega} \\ 0 & x \geqslant \sqrt{\omega} \end{cases}$ et soit ω un nombre tel que l'équation $\sum_{i,j=1}^{k} a_{ij} x_i x_j = \omega$ n'admet pas de solutions entières. Alors, en vertu de ce que

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* La série du second membre de (5), donc aussi de (10) est, dans les conditions α) et β), sommable par la méthode C 1 et sa somme généralisée est égale au premier membre de (5) et (10), respectivement. D'ailleurs il faut tenir compte de ce fait que, bien que pour $k > 1$ la convergence d'une série k-uple n'entraîne pas sa sommabilité par la méthode C 1 (par exemple, si $u_{mn} = 0$, $n > 2$, $u_{m1} = 1$, $u_{m2} = -1$, on a $s_{mn} = 0$, $n > 1$, $s_{m1} = m$, c'est-à-dire $\lim_{m,n} s_{mn} = 0$, tandis que $\sigma_{mn} = {}^1/_{mn} \sum_{1}^{} s_{ij} = \frac{m+1}{2n}$ n'a pas de limite quand m et n tendent vers l'infini), cependant si les deux limites $\lim s_{mn}$ et $\lim \sigma_{mn}$ existent, elles sont égales (voir, par exemple, Lösch [3]).

** Autrement dit: $r_k(n)$ est le coefficient de x^n dans le développement de $\left(\sum_{n=-\infty}^{\infty} x^{n^2}\right)^k = 1 + \sum_{n=1}^{\infty} r_k(n) x^n$. Cette expression pour $r_k(n)$ est valable pour chaque k.

les conditions α) et β) sont évidemment vérifiées, on tire de la formule (10)

$$\sum_{0\leqslant n<\omega} r_A(n) = \frac{(\sqrt{\pi\omega})^k}{\sqrt{D(A)}\,\Gamma\left(\frac{k}{2}+1\right)} + \\ + \frac{(\sqrt{\omega})^{k/2}}{\sqrt{D(A)}} \sum_n{}' \frac{J_{k/2}\left(2\pi\sqrt{\frac{\omega}{D(A)}\sum_{i,j=1}^{k} A_{ij}\,n_i\,n_j}\right)}{\left(\sqrt{\frac{1}{D(A)}\sum_{i,j=1}^{k} A_{ij}\,n_i\,n_j}\right)^{k/2}} \qquad (11)$$

C'est une formule qui donne le nombre $\sum_{0\leqslant n<\omega} (n)$ de points à coordonnées entières à l'intérieur de l'hyperellipsoïde $\sum_{i,j=1}^{k} a_{ij}\,x_i\,x_j \leqslant \omega$ (le signe $'$ dans la somme signifie que la sommation est étendue à toutes les valeurs entières de $n_1, \ldots, n_k$ sauf $n_1 = \ldots = n_k = 0$; le terme de la somme que l'on a considéré à part présente le volume de l'hyperellipsoïde mentionné). La série du second membre est sommable C 1 en vertu du § 2*.

En vertu de la Note à la page 352, la formule (11) reste valable quel que soit ω pourvu que l'on écrit son premier membre dans la forme $\sum_{0\leqslant n\leqslant\omega} r_A(n) - \frac{1}{2} r_A(\omega)$.

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LITTÉRATURE CITÉE

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* Le résultat concernant la sommabilité C 1 de la série du second membre de la formule (11) est surtout essentiel pour $k > 2$ parce qu'après avoir fait une analyse détaillée on trouve la formule

$$\sum_{0\leqslant n<\omega} r_A(n) - \frac{1}{2} r_A(\omega) = \frac{(\sqrt{\pi\omega})^k}{\sqrt{D(A)}\,\Gamma\left(\frac{k}{2}+1\right)} + \\ + \frac{(\sqrt{\omega})^{k/2}}{\sqrt{D(A)}} \sum_{n=1}^{\infty} r_B(n) \left(\sqrt{\frac{D(A)}{n}}\right)^{k/2} J_{\frac{k}{2}}^{k}\left(2\pi\sqrt{\frac{n\,\omega}{D(A)}}\right), \quad \omega > 0,\ k > 0$$

si l'on fait la sommation da la série du second membre par la méthode $\left(R, n, \frac{k-3}{2} + \varepsilon\right)$, $\varepsilon > 0$ pour $k \geqslant 3$, et pour $k = 2$ elle est convergente (voir, par exemple, (4) pour $a_{ij} = \delta_{ij}$).

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ASTRONOMY

THE AGE OF THE EARTH AS DERIVED FROM AN ASTRONOMICAL THEORY

By O. J. SCHMIDT, Member of the Academy

1. At the present time the age of the Earth is determined from the amount of radioactive substances and their decay products in the minerals of the Earth crust. The figures thus obtained lie in the neighbourhood of 2 or 3 milliard years. But, as a matter of fact, these estimates give only the age of the crust, or, strictly speaking, they determine the interval elapsed from the time of the last crystallization. Naturally enough, the Earth itself is older than its crust. Therefore, estimates derived from radioactivity data should be looked upon as the lower limit of the astronomical age. In order to determine the age of the Earth as a celestial body, other methods need to be employed.

The meteoric theory of the origin of the Earth and planets suggested by the author [1] opens a possibility for the evaluation of the astronomical age of the Earth. According to this theory, the Sun once upon a time passed in its galactic revolution through a cloud of obscuring matter, composed of meteorites, dust and gas, and having captured some portion of it went on attended by a meteoric swarm. This swarm shaped itself into a flat lens, and its particles while revolving about the Sun along ellipses are subject to mutual perturbation of orbits. Where their orbits come to cross each other, the smaller particles fall to the larger ones just as meteorites and meteors are known to fall to the Earth down to the present day. This is the process by which planets come into being.

For brevity sake, the word «meteorites» will be used in the sequel to describe all kinds of matter coming down from the swarm to the Earth, irrespective of the size of the particles concerned. For meteors, though burnt in the atmosphere of the Earth, increase this planet's mass by the addition of their own as surely as do meteorites proper in reaching its surface.

2. It is essential for the following to prove that the meteorites that fall on the Earth in our times are members of the solar system rather than newcomers from the distant regions of the interstellar space. This question is known to have been at issue for many a year. Many a writer took part in the discussion, but it will suffice here to mention two names: Hoffmeister [2] who advocated interstellar origin, and Paneth [3] who claimed that meteorites belonged to the solar system on the ground that their chemical composition was much alike to that of the Earth. The last consideration was often called upon to support the idea that meteorites are remains of a disintegrated planet. Yet this argument works as well the other way round, and on the strength of it the planets may be supposed to have arisen from the condensation of meteorites.

Those who held that meteorites come to us from the interstellar space argued from the fact that the tracks left by meteors in the atmosphere seemed to indicate hyperbolic orbits. Yet the accuracy of such a determination of the orbit is open to doubt, the possibility of optical illusion not being excluded. It was not until re-

cently that a meteoric track in the atmosphere was photographed with the aid of two synchronously working motion picture cameras placed at a large distance from each other. The results showed that meteors moved on elliptical orbits, and thus belonged to the solar system. In a summary work published lately Watson (4), referring to this evidence, even puts the question conversely. He asks, Is there any room left for the conjecture that some stray meteors might still prove to be strangers to the solar system?

3. The author's theory does not exclude the possibility of some interstellar dark matter being captured by the Sun even now. But the bulk of meteorites (and meteors) have long been members of the solar system—ever since the Sun crossed the above-mentioned dark cloud. The problem of age determination, if viewed from this angle, is to determine the time of this capture or, what is the same, the start of the accumulation process which we suppose to have built up the Earth. Let us determine the rate of growth of the Earth in this process from the probability of a fall of meteorites to the Earth. That it is possible to use the probability calculus in determining the increment to the mass of the Earth due to meteoric masses has been pointed out by Jeffreys (5) in a general way. Yet he did not think of applying this method to the estimation of the age because he held a different view on the origin of the Earth.

4. For a definite mathematical treatment of the problem we may simplify matters by assuming—which is of course a rather rough approximation—that: a) the Earth brings no dynamic influence to bear upon a meteorite approaching it; b) the meteorites having the two nodes of their orbit located inside the ring between the distances R_1 and R_2 from the Sun in the plane of the ecliptic, will sooner or later collide with the Earth; c) within this ring all positions of the nodes are equiprobable; d) the period of revolution of a single meteorite is substituted with a mean period equal to the period of revolution of the Earth; e) because of interaction between the meteorites the dropping out of some of them is followed immediately by the restoration of the uniform distribution of nodes in the ring; f) a meteorite is bound to collide with the Earth if on its crossing the node the former is just inside the circle which the section of the Earth by the ecliptic fills up at that instant. The possibility of a «grazing» fall on to the part of the Earth projecting above the ecliptic when the node is outside the Earth, is disregarded. How these simplifying assumptions may affect the results of our computations, will be discussed at the end of the paper.

Under these conditions the probability of a node getting inside the circle filled up by the Earth is equal to the ratio of the circle's area πr^2, where r is the radius of the Earth (at the given instant t) to the area of the ring $\pi (R_2^2 - R_1^2)$. And as there are two nodes, the probability of any one of them coming into this position during one revolution of the Earth is $B = \frac{2r^2}{R_2^2 - R_1^2}$.

If Q denotes the initial mass of meteorites in the region alloted to the Earth, and m, the mass already acquired by the Earth at the instant t, the gain during the next year will be the product of the remaining mass $Q - m$ and the above stated probability. Denoting by P the period of revolution of the Earth (one year), the mass acquired during the interval dt is:

$$dm = \frac{2r^2}{R_2^2 - R_1^2} (Q - m) \frac{dt}{P} \qquad (1)$$

Supposing the density of the Earth δ to be constant, we have $m = \frac{4}{3}\pi\delta r^3$, so that r in formula (1) can be expressed through m. If we put $m = Qz^3$, equation (1) transforms into

$$dz = A(1 - z^3)\, dt \qquad (2)$$

where

$$A = \frac{2}{3} \frac{1}{(R_2^2 - R_1^2) P} \left(\frac{4}{3}\pi\delta\right)^{-\frac{2}{3}} Q^{2/3}$$

and does not depend on t. Equation (2) is easy to integrate. If we denote by m_0 and m_T the mass of the Earth at the instants $t=0$ and $t=T$, respectively, the solution of equation (1) will be

$$AT = \frac{1}{6}\log\frac{Q-m_T}{Q-m_0} - \frac{1}{2}\log\frac{Q^{1/3}-m_T^{1/3}}{Q^{1/3}-m_0^{1/3}} + \\ + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2(m_T/Q)^{1/3}+1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2(m_0/Q)^{1/3}+1}{\sqrt{3}} \qquad (3)$$

In order to evaluate the quantity Q it may be expressed through the amount of meteoric matter falling yearly to the Earth at the present time. Let this amount be denoted by Δ. If we then look upon equation (1) as giving the difference for one year, we shall obtain

$$\Delta = \frac{2r_T^2}{R_2^2-R_1^2}(Q-m_T) \qquad (4)$$

5. We can now proceed to numerical values. Put $m_0=0$, which means that we neglect m_0 in comparison with Q, though, of course, the initial («embryonic») mass of the Earth cannot possibly be zero; m_T is taken to be the mass which the Earth has to-day, $m_T=6\cdot 10^{27}$ g; $r_T=6400$ km. The distances R_2 and R_1 should be supposed to indicate regions lying somewhere between the Earth and the Mars, and between the Earth and the Venus, respectively. Let $R_2=1.3$ astr. units; $R_1=0.8$ astr. units. Then we shall have

$$B = \frac{2r_T^2}{R_2^2-R_1^2} = 3.6\cdot 10^{-9}$$

For the value of Δ, or the total mass of the meteorites coming down yearly in our days, different authors have reported different figures. The latest estimate by Watson (4) is about one ton daily. So we put $\Delta = 360$ tons $= 3.6\cdot 10^8$ g. Then $Q-m_T=\frac{\Delta}{B}=10^{17}$ g. This quantity being small in comparison with m_T, we can, when Q is represented in the form

$$Q = m_T\left(1+\frac{\Delta}{Bm_T}\right)$$

neglect the higher, and, whenever possible, even the first powers of the second term in parentheses. Thereby the solution (3) is simplified to

$$AT = -\frac{1}{3}\log\frac{\Delta}{Bm_T} + \frac{1}{2}\log 3 + \frac{1}{\sqrt{3}}\frac{\pi}{6} \qquad (5)$$

Finally we may express A through m_T and r_T, and substitute Q with m_T. Then $A=\frac{B}{3}=1.2\cdot 10^{-9}$. Substituting all these values in (5) we obtain in the end

$$T = 7.6\cdot 10^9 \text{ years.}$$

6. It remains to be realized to what extent the inaccurate determination of certain quantities and the admitted assumptions may have affected the above value.

There is rather a great deal of uncertainty about the true value of Δ. But this quantity enters into formula (5) under the logarithm sign, so that even a strong departure cannot substantially affect the result.

Neither is the latter influenced materially by the arbitrary choice of R_2 and R_1, if only we remain within the reasonable limits imposed by the fact that the ring of the Earth should not be narrower than those of Venus and Mars.

The two nodes have been supposed to be inside the given ring. But a meteorite having only one of its nodes within the ring can also occasionally fall to the Earth. For such meteorites as well as for those having a revolution period longer than that of the Earth, factor 2 in the numerator of the expressions for B and A (the number of intersections of the ring in a year) must be diminished; this means a decrease of A and a respective increase of T.

The neglect of the dynamic influence of the Earth is justified by the fact that owing to the great density of the initial meteoric swarm the meteorites until they closely approach the Earth are not so much controlled by its influence as by their own action upon each other. And when they get near the Earth it is only at very small relative velocities that attraction by the Earth will cause a marked deflection of the orbit. In the case of greater relative velocities (for instance, when the elliptical path of a meteorite intersects the circular path of the Earth at an appreciable angle) the deflection will be quite insignificant. Still allowance for the influence of the Earth will increase the probability B. Accordingly, A will increase, and T diminish.

Finally, a grazing impact acquires some importance only if the horizontal component of the relative velocity is large, *i. e.* if a slightly inclined meteoric orbit has a strong eccentricity. Allowing for it also diminishes T.

None of these factors is as significant as to affect the order of magnitude of T. And the most important of them tend to offset each other.

It appears therefore that the order of magnitude of T is some milliard years, while 7.6 milliard years should merely be regarded as an approximate estimate.

Instead of counting the time from the instant when $m_0=0$, we might have started from any finite value of m. Putting $m_0=\frac{1}{2}m_T$ in order to measure the interval elapsed from the moment when the Earth was half its present mass, we shall find the formula (3) to give a figure of $6.8\cdot10^9$ years, which is but slightly different from the value of T.

Obviously the process of accumulation of matter by the Earth was most violent at the beginning. Its rate fell off rapidly as the store of meteoric matter ran out. At present only very poor remains are left from the meteoric swarm in the region of the Earth. The last two milliard years have but added a layer several centimeters in thickness.

7. Thus, it follows from the author's theory that the Earth started growing as a planet several milliard years ago (by now it is approximately 7.6 milliard years old). This age of the Earth as a celestial body agrees well with the age of its crust as derived from the radioactive decay data (2—3 milliard years), since the crust is certainly younger than the whole planet.

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PHYSICS

ANOMALOUS DISINTEGRATION OF Ag AND Br NUCLEI BY COSMIC RAYS

By A. JDANOV

(Communicated by V. G. Chlopin, Member of the Academy, 14. VI. 1944)

As is known (1), one of the effective methods for the observation of the disintegration of atomic nuclei by cosmic rays is the method of the thick-layer photoplates, suggested and developed in the Radium Institute of the Academy of Sciences of the USSR (2). The tracks of these disintegrations in the thick-layer photoplates mostly have a form of forks with irregular angular distribution («stars»). They correspond chiefly to protons, more seldom to α-particles, and still more seldom to particles with a stronger «ionization power». The yield of nuclear disintegrations at sea level is approximately $8 \cdot 10^{-3}$ disintegrations in hour per 1 cm² of the area of the plate.

This yield rapidly increases with the height, reaching a value 50 times larger at a height of 7000 m(3).

At sea level the greatest percentage of disintegrations corresponds to the triple forks. The forks with 5 tracks are met comparatively seldom, and only with the increase of height can there be observed a slow increase of the number of ejected particles in one disintegration.

In addition to the disintegrations in the form of stars there are also disintegrations in the form of «showers», *i. e.* in the form of forks with a pronounced unilateral direction of tracks (inside a narrow solid angle). Tracks in the showers can belong to protons as well as to mesotrons. In our previous paper(4) we suggested the possibility of registration of slow mesotrons in thick-layer photoplates, and also the estimation of their mass from the average distance between two neighbouring developed grains. One of our papers(5)reported a shower consisting of about 100 heavy particles, which was obtained on a plate exposed at a height of ~9000 m.

In the end of November 1942 we observed an anomalous «yield» of nuclear disintegrations (on several plates this yield at some places was 10^2 times* the normal). We also succeeded then to register a series of showers at sea level.

In many cases these showers may probably be interpreted as a complete disintegration of the nucleus into its components. Such disintegration can be described as «anomalous». The effect has many typical details. For instance, the greatest per cent of disintegrations corresponds in this case to the forks consisting of 4 components. The number of single tracks is noticeably less than the number of disintegrations. In addition to single tracks and to forks consisting of 2—5 tracks, we observed (chiefly in the plates developed in December 1942) a strongly increased** number of disintegrations, many particles being ejected in the form of «stars» or in the form of showers mentioned above.

* We only take into account those disintegrations and single tracks for which the range is more than 10 cm of air equivalent.

** About 2 per cent of the total number of disintegrations.

Below is given the description and results of the measurement of two showers, the photographs of which are shown in Figs. 1 and 2. The first shower consists of 35 tracks, all corresponding to protons, and it is quite natural to attribute it to the anomalous disintegration of the nucleus of Br in the photoemulsion. Measurement of ranges gives us a total energy of protons equal to $\Sigma E_H \cong 80$ MeV.

Assuming that the emitted neutrons have on the average the same energy as protons and taking into account the binding energy of the nucleus (which

Fig. 1

Fig. 2

is given in this region by the formula $\Delta M \cong 8.5\ A$ MeV, where A is the mass number), we obtain for the energy minimum of the incident particle

$$E_N = \sum E_H + \sum E_n + \Delta M \cong 900 \text{ MeV}$$

It is of interest also to consider the momentum of all emitted particles. Measurements show that the sum of projections of momenta of protons on to the axis of shower* is

$$P_H = \sum \sqrt{E_H m_H} \cos \alpha = 17.6 \cong 20$$

Assuming that the total momentum has approximately the same value for neutrons as for protons, we obtain the total value of ~ 40 for all particles from the nucleus. If the momentum $P_N = \sqrt{E_N m_N}$ be computed from the value of the energy of an incident particle, one will obtain ~ 30 in the case of $m_N = 1$. This number is in satisfactory agreement with the value measured from the photographs.

In the second shower there are about 50 tracks, and it is therefore only natural to consider it as the anomalous disintegration of the Ag nucleus. Measurement of the particles of the shower shows that $\sum E_H \cong 150$ MeV, and $P_H \cong 70$. In this case the minimum energy of the incident particle is equal to $E_N \cong 1300$ MeV, and $P_N \cong 35$ (by $m_N = 1$).

The calculations given above show that P_N is in both cases somewhat less than $P_H + P_n$. If we admit that the disintegrations are due to photons, elec-

* The angle between the axis of the first shower and the vertical, $\varphi = 78°$; the angle between the axis of the second shower and the vertical, $\varphi = 81°$.

trons or even to mesotrons, the momentum will have quite a different order of magnitude from the one observed.

In Fig. 3 the energy spectra of protons are shown for both showers. The number of tracks is plotted against the energy in MeV with intervals of 0.5 MeV (position of separate tracks is indicated by points). We can see that the energy maximum in the first shower takes place at 2—2.5 MeV, and in the second shower at 3—3.5 MeV.

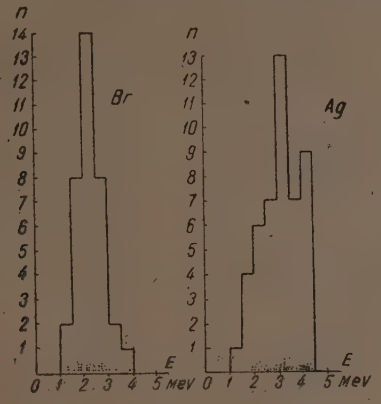

Fig. 3

It is worthy of notice that all the particles in the showers are emitted within a cone whose cross-section is a very eccentric ellipse. In our case, for instance, the axis ratios of the ellipses are 0.13 and 0.15; and for the shower described in the previous paper ([5]) this ratio is 0.1. The strong eccentricity of the disintegration can be explained by the eccentricity of collision between the cosmic particles and the nucleus.

Thus, the existence of the showers indicates the presence of heavy particles with the mass $m \cong 1$ and the energy $\cong 10^9$ MeV in the cosmic radiation at sea level.

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TECHNICAL PHYSICS

ON A STUDY OF THE DETONATION OF CONDENSED EXPLOSIVES

By L. D. LANDAU and K. P. STANIUKOVICH

(Communicated by P. L. Kapitza, Member of the Academy, 29. V. 1944)

The modern hydrodynamic theory of the detonation of condensed explosives developed by Becker (1) and Schmidt (2) is based on the application to the detonation products of the following equation of state:

$$p(v-\alpha)=RT \tag{1}$$

where P is the pressure, v is the volume, T is the temperature, and α is the covolume, representing the incompressible part of the detonation products.

Application of this equation leads to numerous discrepancies, since for a number of explosives it is found that $\alpha \geqslant v_i$ where v_i is the initial volume of the explosive. Various hypotheses on a decrease in α with a rise in pressure, though confirmed by the experimental data of Bridgeman (3) and Schmidt (4), are nevertheless insufficient to determine with accuracy the function $\alpha=\alpha(p)$, and hence do not remove the above discrepancies.

Inasmush as the density of an explosive after it has decomposed is sufficiently high, it is in general rather futile to use any equation of state for a gas, but it is more expedient to compare the decomposition products with some liquid whose particles are in a state of oscillation, thus conditioning the nature of the course of the process of expansion of these products, and only when the detonation products expand to a definite volume v_k, when $v > v_k$, is it possible to make use of the equation of state for an ideal gas

$$pv=RT \tag{2}$$

It is most expedient to build up such elements of the theory of detonation that are based on experimental data determined with a great accuracy, and satisfy all that has been said above. For a number of local action explosives it is found possible to determine the dependence of the detonation velocity D upon the density of the explosive δ_0 over a wide range of values of the latter (from 0.5 to 1.8). This dependence is shown on the accompanying graph and may be approximated by the simple formula

$$D=B\delta_0^{\alpha} \tag{3}$$

Making use of the fundamental equations of the hydrodynamic theory of detonation

$$E=Q+\frac{p+p_0}{2}(v_0-v) \tag{4}$$

$$p-p_0=-\frac{\partial p}{\partial v}(v_0-v) \tag{5}$$

$$D^2=-v_0^2\frac{\partial p}{\partial v} \tag{6}$$

where E is the energy at the front of the detonation wave and Q is the heat of reaction, it may be shown that the following expression is true:

$$\frac{d \ln D}{d \ln v_0} = \frac{2v - v_0}{2(v - v_0)} + \frac{v_0 \frac{\partial p}{\partial T}}{2\left(2c_v + (v - v_0)\frac{\partial p}{\partial T}\right)} \tag{7}$$

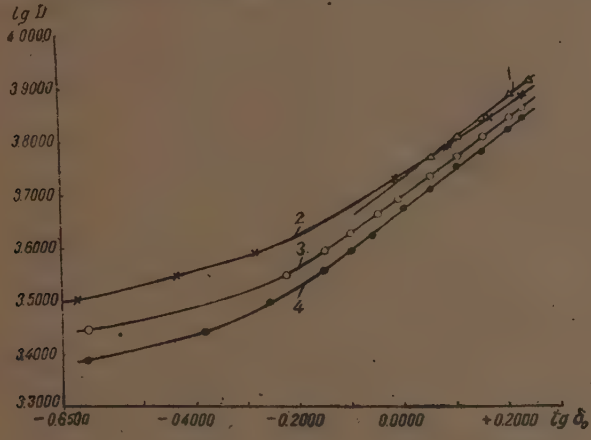
Since $v_0 = 1/\delta_0$, we get from (3)

$$\frac{d \ln D}{d \ln v_0} = -a \tag{7a}$$

According to our hypothesis in which we assume that the free energy of the detonation products F depends upon the vibration frequency of their molecules, we may write:

$$p = p_0 + LkT \frac{\partial \ln \omega}{\partial v} \tag{8}$$

where p_0 is that part of the pressure which is independent of the temperature, L is the number of degrees of freedom for the molecules moving as a whole,



1, ten; *2*, tetryl; *3*, picric acid; *4*, trinitrotoluene

k is the Boltzmann constant, and ω is the frequency of vibrations. Since $\omega \approx c/r$, where c is the velocity of sound in the medium considered and r is the distance between the molecules, then inasmuch as c is a power function of the volume (which will become clear from what follows) and $r \approx v^{1/3}$, the equation of state may be written in the following form:

$$p = \bar{p} + \frac{AT}{v} \tag{9}$$

where $\bar{p}$ is the part of the pressure independent of the temperature, and A is a certain coefficient depending upon the number of degrees of freedom. It follows from (9) that $\partial p/\partial T = A/v$, and hence (7) may be rewritten in the form

$$-a = \frac{2v - v_0}{2(v - v_0)} + \frac{v_0}{2\left[2\frac{c_0}{A}v + (v - v_0)\right]} \tag{7b}$$

whence it follows that $v \approx v_0$. And upon assuming on the basis of equation (9) that in the region considered

$$pv^n = \text{const} \tag{10}$$

instead of the analogous equation for an ideal gas

$$pv^\gamma = \text{const} \tag{10a}$$

this will signify that $c \approx v^{\frac{1-n}{2}}$. The value of n, as will be shown, may be determined from (7b). Indeed, on the basis of the classical theory of detonation and making use of equation (10) we may write

$$v_i = \frac{n}{n+1} v_0 \tag{11}$$

$$p_i v_0 = D u_i \tag{11a}$$

$$u_i = \frac{D}{n+1} \tag{11b}$$

$$c_i = \frac{n}{n+1} D \tag{11c}$$

where p_i is the pressure at the front of the detonation wave, c_i is the velocity of sound, and u_i is the velocity of the medium behind the detonation wave front; whence it follows that

$$a = \frac{n-1}{2} - \frac{n+1}{2\left[2n\frac{c_v}{A} - 1\right]} \tag{7c}$$

inasmuch as c_v and A depend upon the number of degrees of freedom, *i. e.* we may write

$c_v = 12$, $A = 5n - 3$, for diatomic molecule
$c_v = 18$, $A = 6n - 3$, a simple system for triatomic molecules
$c_v = 12$, $A = 5n - 3$, a linear system » » »

Thus, it is evident that by employing the method of successive approximations we may determine the value of n from equation (7c) as well. Now it becomes possible to determine the coordinates of the point where equation (10) is conjugate to equation (10a).

With this aim in view let us determine the work of expansion w of the detonation products when the volume changes from v_i to v_k.

$$w = \frac{p_i v_i}{n-1} - \frac{p_k v_k}{n-1} + \frac{p_k v_k}{\gamma - 1} = E \tag{12}$$

Making use of equations (11), (11a), (11b), (11c) and disregarding the quantity $p_k v_k/(n-1)$ which is small in comparison with $p_k v_k/(\gamma-1)$, as well as the change in c_v with T, we get

$$\frac{p_k v_k}{\gamma - 1} = Q - \frac{D^2}{2(n^2-1)} \tag{12a}$$

or, denoting $Q - \frac{D^2}{2(n^2-1)}$ by ΔQ

$$c_v T_* = \Delta Q \tag{12b}$$

The most interesting parameters computed for several explosives are:

Explosives	$Q\frac{\text{cal}}{\text{degree}}$	D m/sec	δ_0	a	n	u_i m/sec	c_i m/sec	δ_i	p_i atm.	ΔQ	p_k atm.	δ_k	T
Nitrotoluene	950	7000	1.60	0.78	3.2	1680	5320	2.10	190000	320	2450	0.52	1580
Tetryl	1150	7600	1.63	0.76	3.1	1850	5750	2.15	230000	350	2000	0.46	1540
Picric acid	1000	7100	1.63	0.76	3.1	1730	5370	2.15	210000	300	2360	0.49	1530
Ten	1400	8400	1.69	0.80	3.2	2000	6400	2.28	275000	490	2900	0.52	1940

For local action explosives it is quite allowable to use an approximate value of $n = 3$, which greatly simplifies all the calculations without any great sacrifice in accuracy.

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PHYSICAL CHEMISTRY

ELECTRONIC THEORY OF SURFACE TENSION IN METALS

By A. G. SAMOYLOVICH

(Communicated by A. N. Frumkin, Member of the Academy, 1. VII. 1944)

Of the theories of surface tension in metals that have heretofore appeared in literature no one takes due account of forms of non-electrostatic origin upon which depends the equilibrium of the system. However, the stresses caused by these forces make up an essential part of the surface tension.

In this respect the recently published paper by Dorfman (1) is not an exception. In it surface tension is defined as the electrostatic energy of a double layer of metal

$$\sigma = \frac{1}{8\pi} \int_{-\infty}^{+\infty} E^2 \, dx \qquad (1)$$

This definition, as will be shown later, contradicts the conditions of equilibrium.

The present paper is an attempt to show how the general theory of capillary forces, as developed by Van-der-Waals, Hulshof and especially by Bakker (2), may be brought into harmony with the modern conceptions of the electronic theory of metals. It has been shown by Bakker in a general way that surface tension is representable in the form

$$\sigma = \int (P_N - P_T) \, dx \qquad (2)$$

where P_N is external pressure, and P_T is pressure in a plane parallel to capillary layer (the x axis is defined to be directed perpendicular to capillary layer).

The quantity

$$\sigma_x = P_N - P_T \qquad (3)$$

which will be described here as differential tension, means that $\sigma_x \, dx$ is the stress which a boundary perpendicular to the capillary layer exerts by its portion $x; x + dx$.

Let us consider a rough model of a metal as follows. The totality of ions will be treated as an incompressible fluid in which a positive electric charge is distributed continuously with a constant density ρ_0. The electronic gas will be looked upon as a Thomas-Fermi-Weizsäcker approximation. For this model the energy of the metal can be written down in the following form

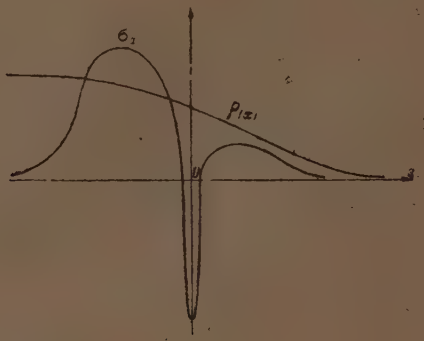
$$U = \int \left[\frac{3h^2}{40m} \left(\frac{3}{\pi} \right)^{2/3} \rho^{5/3} - \frac{3}{4} \left(\frac{3}{\pi} \right)^{1/3} e^2 \rho^{4/3} + \frac{h^2}{32\pi^2 m} \frac{1}{\rho} (\nabla \rho)^2 + \frac{1}{8\pi} E^2 \right] dx \qquad (4)$$

Varying this expression according to the well-known rules, we find

$$\left.\begin{aligned} P_N &= \Pi + \frac{h^2}{16\pi^2 m}\left[\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x}\right)^2 - \frac{\partial^2\rho}{\partial x^2}\right] - \frac{E^2}{8\pi} \\ P_T &= \Pi + \frac{E^2}{8\pi} \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} \Pi &= \frac{h^2}{20m}\left(\frac{3}{\pi}\right)^{2/3}\rho^{5/3} - \frac{1}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2\rho^{4/3} \\ &\text{outside of the metal,} \\ &= \frac{h^2}{20m}\left(\frac{3}{\pi}\right)^{2/3}\rho^{5/3} - \frac{1}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2\rho^{4/3} - \rho_0\varphi + u_0 \\ &\text{inside the metal.} \end{aligned}\right\} \tag{6}$$



Here φ is potentional, and u_0 is density of energy with uniform distribution of electrons, the condition of equilibrium in this case being

$$P_N = 0 \tag{7}$$

From this, in accordance with (3) and (2), we obtain

$$\sigma_x = \frac{h^2}{16\pi^2 m}\left[\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x}\right)^2 - \frac{\partial^2\rho}{\partial x^2}\right] - \frac{E^2}{4\pi} \tag{8}$$

$$\sigma = \int_{-\infty}^{+\infty}\left[\frac{h^2}{16\pi^2 m}\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x}\right)^2 - \frac{E^2}{4\pi}\right]dx \tag{9}$$

To facilitate numerical calculations, the density distribution of the electronic gas was supposed, as an approximation, to obey the formula

$$\rho = \rho_0 \frac{1}{e^{x/\lambda} + 1} \tag{10}$$

where λ is a variational parameter obtained by minimizing (4). This distribution reproduces with sufficient accuracy the one which Bardeen [3] found for sodium by a numerical integration of Hartree-Fock's equation. The computed results are given against experimental data in the table. A figure brings out the stress distribution in the capillary layer.

The conclusions to be drawn from what has been said here are these.

1. The value of surface tension in metals can be properly estimated as to its order of magnitude, by means of the electronic theory, without having recourse to empirical constants.

	σ dyne/cm		V eV	E_m eV	Φ eV
	after formula (9)	experiment			
Na	400	294	6.33	3.16	2.25
K	227	411	1.75	2.06	2.24
Cu	1 285	1 103	6.10	7.10	4.74
Ag	991	800	3.95	5.52	4.82
Hg	700	465—547	5.93		

2. In order to build up an adequate theory of surface tension in metals one has to take into account non-electrostatic forces. An important rôle is played here by the anisotropic part of the stresses. If the electrostatic forces be counterbalanced by the forces giving rise to anisotropic tensor, we should then have a negative surface tension. This actually occurs in electrocapillary phenomena when the electrostatic forces in the double diffuse layer are counterbalanced by anisotropic osmotic pressure.

3. As is seen from the figure, the capillary forces are largely concentrated within the metal. This refutes Dorfman's contention that surface tension of metals is connected with the presence of an electronic cloud over the surface.

4. It is evident from (8) that the external work of the electrostatic forces is negative. When the electrostatic energy of the double layer increases with the surface area, it does so at the cost of the kinetic and exchange energies of the electronic gas.

The potential jump V at the metal/vacuum boundary and the capacity of the double layer at the surface of the metal can be evaluated from the distribution (10). If the data listed in the table be compared with Fermi energy maximum of an electron E_m and with the work function Φ, it will be found that $V < E_m$ for all of the tabulated metals but sodium (the accurate computations done by Bardeen have shown this to be true for sodium as well). So Wigner and Bardeen [4] appear to be right in believing that the presence of a double layer cannot possibly account for the work function. The conflicting result obtained by Dorfman is based on the relation between surface tension and the jump in potential, as derived from the fallaceous formula (1).

For all of the tabulated metals the capacity of the double layer is a magnitude of the order of 4 μF/cm². The order of magnitude of the capacity of the double layer at the metal/solution-of-electrolyte boundary, as determined from the electrocapillary curve, is 20—50 μF/cm². It will be obvious that the main part in electrocapillarity is played by the double layer in electrolyte.

The theory put forward here admits of generalization in different directions. A similar theory, for instance, can be evolved in the Hartree-Fock approximation.

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PHYSICAL CHEMISTRY

OXIDATION OF HYDROCARBONS IN THE GASEOUS PHASE

II. OXIDATION OF ETHANE AT TEMPERATURES ABOVE 600° C

By N. CHIRKOV

(Communicated by N. N. Semenov, Member of the Academy, 21. IV. 1944)

The author has shown elsewhere (1) that at 466° and 486° C water vapour catalyses the oxidation of ethane. He has also described this catalysing action quantitatively by means of an empirical formula. Further research revealed that the catalysing action of water vapour ceases at higher temperatures.

Insvestigating the dependence of the induction period of the explosion upon the pressure with and without additions of water vapour it was found that when

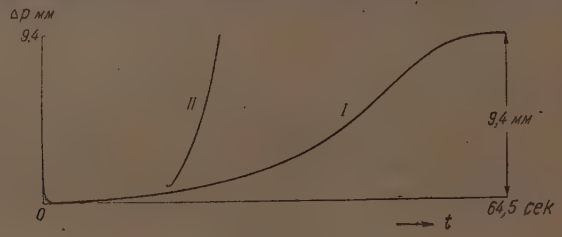

Fig. 1

$T < 600°$ C the addition of water vapour curtails the induction period. The relative shortening of the induction period is the smaller, the higher the temperature.

Lastly, when the temperature was raised to 630° C, in the pressure range 100—300 mm of Hg practically no difference was observed between the induction periods of a pure mixture and one to which 3.6 per cent H_2O was added. At these temperatures ($T \geqslant 600°$ C) the reaction kinetics as well differs from that at $T = 450$—500° C, as will be shown below.

As previously, the kinetics was studied by following up the change in pressure in the reacting mixture. The change in pressure with time was measured with a membrane manometer. The ray reflected from the manometer mirror impinged upon a photographic film rolled on a reel in a photoregistering device.

The use of a photographic method for registering the reaction velocity was necessitated by the fact that at $T \cong 600°$ C the reaction proceeds rather rapidly.

A representative photograph of an experiment at $T = 604°$ C is given in Fig. 1, on which two processes have been recorded: one at $p = 40$ mm, and another (explosion) at $p = 81$ mm.

The sensitivity of a membrane manometer usually decreases with the increase of the membrane deflections. Therefore a photograph gives a somewhat distorted picture of the kinetic curves. For our purposes, however, a membrane was selected

whose deflection determined by the ordinate of the curve on a photograph was practically proportional to the change in pressure of the reacting gases (thus, at the start of the curve (in Fig. 1) 1 cm along the ordinate corresponds to 2.8 mm of Hg, and at the end of the curve, to 3.2 mm of Hg). Hence, a photograph reproduces a kinetic curve that lies close to the true one. To obtain the true kinetic curves a calibration curve was obtained for the membrane manometer, with the aid of which it was possible to express the values measured along the ordinate in terms of mm of Hg.

Knowing the speed of rotation of the reel to be $\frac{248}{87}$ mm/sec. for the curves in Fig. 1, it was not difficult to determine the time t for any Δp.

In Fig. 2 are given true kinetic curves for $T = 604°$ C and $p_I = 60$ mm, $p_{II} = 40$ mm. At first glance they do not seem to differ much from the corres-

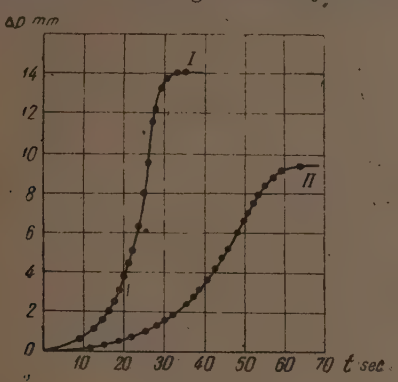

Fig. 2

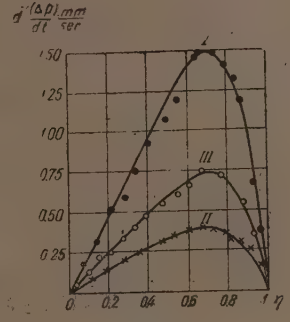

Fig. 3

ponding curves obtained for $T = 466°$ C [1]. However, if we plot the dependence of the reaction rate $d(\Delta p)/dt$ (which we were able to determine sufficiently accurately from the kinetic curves Δp—t) upon the quantity of the substance that has reacted, η, we find quite essential differences (Fig. 3). The high-temperature curves are either sligthly convex or nearly straight in the region of small η, whereas the low temperature curves are quite concave [1]. Quantitatively this means that in the first case the reaction rate at small η is proportional to η, and in the second, to η^2.

The entire course of the curves in Fig. 3 is well described by the equation

$$d\eta/dt = k\,(e^{b'\eta} - 1)\,(1 - \eta) \qquad (1)$$

in which the constants b' and k have the following values, respectively: 3.5 and 0.47 for Curve I; 2.5 and 0.28 for Curve II; and 3 and 0.35 for Curve III, if the rate is expressed in mm/sec. Comparison of equation (1) with equation (2) describing the process at low temperatures [1] discloses the absence of the factor η in equation (1), which, as we have elucidated, characterizes the catalysing action of water. The fact that this factor does not figure in the equation is fully comprehensible, since at $T = 604°$ C water does not behave as a catalyst. Equation (1) may be derived theoretically on the basis of the concept of positive interaction of chains [2].

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MINERALOGY

SERPIERITE AND CYANOTRICHITE FROM KAZAKHSTAN

By F. V. CHUKHROV

(Communicated by A. E. Fersman, Member of the Academy, 9. VI. 1944)

Serpierite [5(Cu, Zn, Ca) $0 \cdot 2SO_3 \cdot 6H_2O$] is one of the very rare minerals of the oxidation zone of sulphide deposits. Until now it has been found only in Laurion (Greece). In the course of a study of mineralogy of the oxidation zone of Central Kazakhstan, the mineral was discovered by the present author in the Akchagyl deposit, situated within about 35 km east of the Mointy station, on the Karaganda railway.

The mineralization within the deposit is confined to skarns formed after limestones. The primary sulphide minerals are represented mainly by galena, sphalerite, chalcopyrite. Among hypergene minerals, the following have been discovered: covellite, chalcocite, brochantite, linarite, serpierite, gypsum, chrysocolla, cerussite, smithsonite, calamine, atacamite *etc.*

Serpierite belongs to minerals rather widespread in the Akchagyl deposit. Segregations of serpierite are observed in the weathered ore or in cracks within the limestone, immediately adjoining the ore body. The most common mode of occurrence of serpierite are incrustations up to 1 mm thick, as well as tubercular, more of less semispherical aggregates. The structure of the crusts and tubercular aggregates is generally a radial one. Occasionally, indistinctly developed laminar small crystals and crystal concrescences are encountered. Of the minerals associated with serpierite, the following should be mentioned: linarite, brochantite, gypsum, cerussite, smithsonite and calamine. The most frequently serpierite occurs together with linarite. With respect to gypsum and linarite, serpierite is an earlier mineral.

Under the microscope serpierite is usually seen as radial aggregates. A distinct pleochroism is revealed: light green following the elongation, and nearly colourless, perpendicular to the elongation.

The extinction is straight with respect to elongation and cleavage planes. Elongation, positive. Optic sign, negative. Optic angle, as measured on the Fedorov stage, 33°. $N_g = 1.650$; $N_m = 1.644$.

Incomplete qualitative analysis went to confirm the identification of the mineral.

The earlier formation of serpierite as compared to calamine, recorded for Akchagyl, and its later formation as compared to smithsonite, characteristic of the serpierite from Laurion, points to the formation of this mineral at a comparatively late, but not final stage in the weathering of ores.

Cyanotrichite ($4CuO \cdot Al_2O_3 \cdot SO_3 \cdot 8H_2O$) is a mineral more widespread than serpierite, although it has been recorded from a very limited number of deposits. In the USSR P. P. Pilipenko was the first to record it in the Chaghirsky mine, Western Altai. In 1938 this mineral was discovered by the present author in the Berkara deposit (Central Kazakhstan) and in the Berezovsky deposit in Western Altai.

1. The Berkara deposit is located in the Karaganda region, 80 km south-west of the town of Karkaralinsk, in the spurs of the Berkara mountains. The mineralization is found in sandstones, marlaceous shales and hornstones. The ores are of a polymetallic nature. The chief primary sulphide minerals in the deposit are pyrite, sphalerite, galena and chalcopyrite. In the zone of hypergenesis the following minerals are found: native copper, chalcocite, covellite, cuprite, atacamite, brochantite, jarosite, linarite, cyanotrichite, *etc.*

The cyanotrichite is comparatively widespread in the deposit. Characteristic of its segregations is the presence of incrustations, composed of radial-fibrous, less frequently asteriated aggregates. The colour of the mineral aggregates is an intense blue to a light blue. The most typical is the association of cyanotrichite with brochantite, the latter being evidently replaced by the former.

In addition, associated with cyanotrichite are linarite, malachite, azurite. Under the microscope the fibres of cyanotrichite are seen to be collected in bundles and partly in spherulites. Elongation, positive. Extinction, straight with respect to elongation. Pleochroism distinct: in the direction of elongation it is light blue: and perpendicular to elongation, nearly colourless. The refractive indices of the cyanotrichite from Berkara correspond to data, cited in the text-book of Larsen and Berman.

A carefully selected cyanotrichite was analysed by F. J. Anossov. The results of analysis are tabulated below.

Component	Content % from analysis	Molecular amount	Theoretical composition
CuO	48.30	0.607	49.3
CaO	traces	—	—
MgO	0.00	—	—
Al_2O_3	15.69	0.153	15.9
SO_3	12.25	0.153	12.4
H_2O	22.05	1.225	22.4
SiO_2	1.98	—	—
	100.27		100.0

The ratio of molecular quantities for cyanotrichite from Berkara ($3.97\ CuO : Al_2O_3 : SO_3 : 8H_2O$) corresponds very closely to the formula of this mineral.

2. The Berezovsky deposit is located in the upper course of the Berezovka River, representing the right-hand side tributary of the Krasnoyarka River (Predgornensky region, East Kazakhstan). The ore deposit is considered as an interbedded deposit. The enclosing rocks are carbonaceous phyllite shales, limestones, sericitic quartz schists and porphyrites. The sulphide minerals in the deposit are represented by chalcopyrite, sphalerite, galena, pyrite, *etc.* In the oxidation zone the following minerals have been encountered: native copper, cuprite, aurichalcite, cerussite, malachite, brochantite, cyanotrichite, *etc.*

Cyanotrichite has been found by the author from the drift of the 4th horizon. The segregations of cyanotrichite are represented by crusts on the ferruginized wall rock and on brochantite aggregates. The crusts are composed of acicular or capillary crystalline individuals, forming groups and bunches. Both are characterized by a radial orientation of the crystalline individuals. The colour of the mineral varies from an intense blue to bluish-white in aggregates composed of the finest capillary individuals.

In its qualitative reactions and optic properties the cyanotrichite from the Berezovsky deposit does not differ from that from Berkara deposit. The paragenetic relations with brochantite show that cyanotrichite belongs to the relatively late minerals of the oxidation zone. To judge by the specimens in the museum collection, at Nizhni Taghil cyanotrichite was formed later than malachite.

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CYTOLOGY

ON THE TREATMENT OF VEGETABLE CELLS WITH COLCHICINE

By N. N. KARTASHOVA

(Communicated by A. A. Zavarzin, Member of the Academy, 9. VI. 1944)

It was suggested by Koltzoff (1) in 1939 that colchicine should increase the viscosity of the cytoplasm.

Frey and Parks (2) have demonstrated in their excellent study the relation between the variation in cytoplasm viscosity and the stages of mitosis. According to these authors, the whole process of spindle formation at prophase and at early metaphase occurs with minimum cytoplasm viscosity. Since the later half of the metaphase and during the anaphase there is a sharp increase in cytoplasm viscosity. One should think, therefore, that a change in cytoplasm viscosity may be the main obstacle to the formation of achromatic spindle in cells which have been treated with narcotica, high temperature or with a solution of colchicine. We have conducted a series of special experiments on the influence of colchicine upon cytoplasm viscosity in plant cells.

As a material for these experiments served the stems of tomato (*Solanum Lycopersicum*) and of the orpine (*Sedum* sp.), as well as the petioles of young leaves of *Aegopodium podagraria*, *Primula obconica* and *Pelargonium zonale*. The experiments were carried out in May.

Determinations of cytoplasm viscosity were made with the aid of the centrifuging method introduced by Heilbrunn and worked out by F. Weber. Petioles and stems were submersed in 0.1 per cent colchicine solution for varying length of time. Corresponding organs of the same plants immersed in distilled water for the same period served as control. Experimental and control plants were centrifuged simultaneously; after centrifuging sections of the cortex of the stems and of the petioles were made by hand and studied under the microscope at a magnification of 400 diameters. The degree of cytoplasm viscosity was estimated by the displacement of the plastids.

In each experiment the distribution of the plastids along the greater axis of the cell was determined in ten cells with the aid of an eye-piece net.

The accompanying table shows the distribution of the chloroplasts after centrifuging along the greater diameter of the cell, measured in the divisions of the eye-piece net. Zero stands for the absence of plastids at a certain point of the cell. Other figures show the numbers of the plastids present. The symbol × shows that the mass of the plastids has been thrown to the bottom of the cell, which made the counts difficult.

The experiments with the orpine were the most extensive ones. Stems of this plant were subjected to the action of colchicine for 20 minutes, 1, 2, 4, 6, 12 and 24 hours.

Centrifuging lasted 2 minutes at 2 000 revolutions per minute. The first three samples (30 minutes, 1 hour and 2 hrs) did not show any differences as compared with the control. Microscopic examination yielded rather inconsistent results.

Distribution of Plastids in the Cells of the Stem of *Sedum* after Centrifuging

Distribution of the plastids along the greater diameter of the cell / Cell No.	6-hour-long exposure to colchicine										Control									
Divisions of the eye-piece net	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	0	0	0	2	0	0	2	0	4	0	0	0	0	0	0	0	0	0	0	0
2	4	0	2	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
3	10	3	0	0	2	0	2	2	1	0	0	0	0	0	0	0	0	0	0	0
4	1	1	2	0	1	0	0	1	2	1	0	0	0	0	0	0	0	0	0	0
5	0	0	1	15	3	0	1	0	1	2	0	0	0	0	0	0	0	0	0	0
6	1	2	0	6	1	1	2	2	0	3	0	0	0	0	0	0	0	0	0	0
7	×	×	5	0	3	3	1	3	2	×	×	0	1	0	0	0	0	0	0	1
8		×	×	2	0	2	2	8	5			×	17	0	0	0	0	0	0	1
9			×	3	3	1	1	×	×					×	0	0	0	0	0	0
10					2	0	×	6	×						×	0	0	0	0	0
11					×	×	×	6								×	0	×	×	×
12																	×		×	×
13																				

In some cells both from experiment and control, the plastids were gathered at the «bottom», in others they were evenly distributed throughout the cell.

After a four-hour long colchicine treatment there appeared a decisive difference between experiment and control. In all the plants of the control nearly all the plastids (save for two or three) were displaced to the bottom; in the experimental material the falling of the plastids to the bottom of the cell, though occasionally observed, never attained such a degree. A still sharper difference was observed after a six-hour-long treatment.

It should be deduced from the table, therefore, that cytoplasm viscosity was higher in the treated plants than in the control ones.

In those cells which were subjected to the action of colchicine, about 25 per cent of the plastids were scattered all over the area of the cell, whereas in the cells of the controls all the plastids were displaced to the bottom, the strength of centrifuging being the same in the two cases. With 12-hour-long exposure to colchicine solution the cytoplasm viscosity still increased, whereas after 24-hour-long treatment there was observed no displacement of the plastids in the centrifuged cells of the experimental plants.

It should be noted that a gradual increase in viscosity was observed during the period of the experiments in the cells of the control plants, although it was considerably less than in the experimental material.

The second experiment was made with *Primula*. The plants were subjected to the action of colchicine dirung 6. 12 and 24 hours. Shorter exposures were not used, since experiments with *Sedum* have shown that colchicine in the concentration used in our experiments, produced no pronounced effect upon the cytoplasm viscosity until the treatment is made to last 6 hours.

Cytoplasm viscosity proved in general to be higher in *Primula* than in *Sedum*. Displacement of chloroplasts within the cells of the control plants was attainable only after a 5-minute-long centrifuging at 2500 revolutions per minute; such were the conditions of centrifuging applied in all the experiments with this plant.

With all the exposures studied total sedimentation of the plastids to the bottom of the cells was observed in the control only. Even in the case of six-hour-long treatment with colchicine cytoplasm viscosity was increased to such an extent that the chloroplasts were displaced by no more than half the length of the cell.

With a 12-hour-long exposure the chloroplasts were rather evenly distributed over the cell, and in its lower quarter only there could be observed a certain concentration of the plastids. After a 24-hour-long treatment with colchicine there was

such a strong increase in cytoplasm viscosity, that centrifuging failed to produce any effect whatever.

In the control all the chloroplasts appeared to be thrown down to the bottom.

Analogous results were obtained with *Pelargonium*, *Aegopodium* and *Solanum*. After a 24-hour-long treatment with colchicine solution the chloroplasts appeared after centrifuging to retain their original positions. On the other hand, in the control material which was kept during the same time in distilled water, all the chloroplasts invariably appeared to be displaced after centrifuging so as to assume the lowest position within the cell. Special experiments have shown that even weak colchicine solutions caused a noticeable increase in cytoplasm viscosity of plant cells.

From the above data we find it possible to assume that this very change taking place in the cytoplasm is one of the causes of the absence of achromatic figures in the cells of plants treated with colchicine.

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PLANT PHYSIOLOGY

INFLUENCE OF GRAFTING UPON THE GENERATIVE SPHERE IN COTTON-PLANT

By A. I. ZHURBIN

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The problem of the degree and kind of the mutual interaction between the components of a graft stands in need for further study. We find it, therefore, important to study the effect of the stock upon the generative sphere of the scion.

Our aim was to study the influence of the stock upon the sterility of the scion and suggest a way of overcoming it. As a material for our experiments we have chosen sterile and slightly fertile forms of cotton. The following features were studied: time of the onset of flowering, intensity of flowering, dropping of ovaries, fertility of anthers, fertility of ovules, number of mature bolls and seeds.

We were interested not only in the strength of the influence of the stock upon the generative sphere of the scion, but also in the dependence of this influence on the greater or lesser genetic affinity between them. In accordance, we have chosen as stocks a series of cotton species differently related to the scion. Thus, in the one instance there was selected the nearly sterile triploid which had been obtained by us [1] by crossing *Gossypium hirsutum* L. 0100 with *G. barbadense* L. 35-1; the following forms served as stocks for grafting: 1) *G. barbadense* L. 35-1, *i. e.* the form which was the male parent of the scion; 2) *G. barbadense* L. 35-2, *i. e.* a somewhat more remote form; and 3) *G. hirsutum* L. 1306 × *G. barbadense* L. v. *maritima*, *i. e.* a hybrid between varieties more remote still. In another instance a slightly fertile amphidiploid was chosen for scions; it was produced by us in 1939 [1] from a myxoploid *G. hirsutum* L. 36M2 × *G. barbadense* L. 35-1. The following forms were used as stocks: 1) *G. hirsutum* L. 36M2, *i. e.* the form which was the mother form in breeding the scion; 2) *G. hirsutum* L. 13638 and 8517, *i. e.* two other varieties which were more remotely related to the scion; and 3) *G. hirsutum* L. 1306 × *G. barbadense* L. v. *maritima*, *i. e.* a hybrid between varieties more remote still.

All the stocks were forming flower-buds toward the moment of grafting. The scions were not ready to bloom. In order to be sure that the scions were supplied with food only from the roots and leaves of the stocks, there were left on the average ten leaves on each stock, while all the leaves were removed from the scions. In addition, all the flower-buds and shoots were removed from the stock, and the sympodia decapitated. After 40 days following grafting the scion has on the average as many leaves as there were on the stocks; however, the leaves were smaller, and there were small leaf-bearing shoots, in addition. After 60 days following grafting there were on the average only three leaves on each stock, while the scions have developed a great leaf-area. During all the time from the setting of the flower-buds until blooming, *i. e.* during the period of the formation of the sex cells (which,

according to our observations, takes in *G. hirsutum* L. 0100 from 20 to 23 days, from the setting of the archesporial cells to mature pollen grains and embryo sacs), the scions received mineral and organic food, as well as hormones and other highly physiologically active substances exclusively or predominantly from the stocks.

The fertility of the anthers was determined by recording flowers with mature and pollen-shedding anthers. Fertility of the ovules was determined by recording ovaries in which ovules were developed. A developing ovule differs from a dying one by its considerable dimensions and white coloration, while the dying ovule never exceeded 1—2 mm in diameter and was dark-grey in colour, wrinkled and drying up. Ovules were recorded both in all the dropping ovaries and in the boll, when ripe.

Blooming of the scions began at different intervals upon grafting.

Table 1

Scion	Stock	Grafting	Beginning of blooming	Number of days from grafting to blooming
Triploid *G. hirs.* 0100×*G. barb.* 35-1	*G. barb.* 35-1	8.VII	8.IX	62
	G. barb. 35-2	2.VII	22.VIII	52
	»	9.VII	27.VIII	50
	»	8.VII	20.VIII	44
	G. hirs. 1306×*G. barb.* v. *marit.*	15.VII	25.VIII	43

The scion grafted to the hybrid stock which was genetically the most remote, was the first to begin blooming. Then, after a period from one to nine days (Table 1) there began to bloom scions grafted to the nearer related stocks; finally, after 10—18 days began blooming the scions grafted to the most closely related stocks. Grafting of the amphidiploid yielded analogous results.

The intensity of blooming is represented by the average number of flowers which blossomed within 10 days (Table 2). Blooming was observed from the end of August and beginning of September to the end of September. The intensity of blooming was the same in all the scions both at the beginning and the end of September.

Table 2

Scion	Stock	Intensity of blooming
Triploid *G. hirs.* 0100×*G. barb.* 35-1	*G. barb.* 35-1	5
	G. barb. 35-2	7
	G. hirs. 1306×*G. barb.* v. *marit.*	12
Amphidiploid *G. hirs.* 36M2×*G. barb.* 35-1	*G. hirs.* 36M2	5
	G. hirs. 8517 and 13638	6
	G. hirs. 1306×*G. barb.* v. *marit.*	7

The intensity of the blooming of the scions varied according to the kind of the stocks, *viz.* it was higher in genetically remote stocks.

The process of shedding of ovaries in the scions as dependent on the kind of stock is shown in Table 3 representing the number of ovaries (in per cent to the total number) remaining during various periods after blooming.

The data contained in Table 3 show that grafting has considerably reduced the shedding of ovaries. It should be noted that the reduction is the more expressed, the more remote is the genetic relation between the scions and stocks. As to food

Table 3

Scion	Stock	Number of ovaries recorded	Per cent of the ovaries remaining within different periods after blooming (days)					
			10	20	30	40	50	60
Triploid	none	172	58	44	27	26	25	24
G. hirs. 0100×*G. barb.*	*G. barb.* 35-1	6	100	50	50	33	33	33
35-1	*G. barb.* 35-2	44	100	82	59	54	54	52
	G. hirs. 1306×*G. barb.* v. *marit.*	24	100	83	66	66	66	66
Amphidiploid	none	22	73	59	41	41	41	41
G. hirs. 36M2×*G. barb.*	*G. hirs.* 36M2	10	100	90	80	80	80	60
35-1	*G. hirs.* 1306×*G. barb.* v. *marit.*	10	100	100	100	100	90	90
	G. hirs. 8517 and 13638	28	100	100	100	97	97	97
G. barb. 35-1×*G. Thurberia* Tod.	none	hundreds	0	0	0	0	0	0
	G. barb. 35-2	16	87	87	87	87	87	87

and water supply, all the scions were in equal position, so that differences in the degree of shedding could not be attributed to differences in food or water supply. Evidently some other agents are responsible for the shedding of ovaries by the scion; they should be looked for in the stock, in the specific peculiarities of its nutrition.

The fertility of the anthers and ovules is represented in Table 4. All the flowers obtained were divided into two categories: those containing from 10 to 50 per cent mature and pollen-shedding anthers, and those with underdeveloped anthers.

Table 4

Scion	Stock	Per cent of flowers with developed anthers	Number of flowers studied	Per cent of ovaries with developing ovules	Number of ovaries studied
Triploid	none	2.5	406	76	136
G. hirs. 0100×	*G. hirs.* 1306×*G. barb.* v. *marit.*	2.7	37	100	8
×*G. barb.* 35-1	*G. barb.* 35-2	8.6	68	100	25
	G. barb 35-1	11.0	9	100	4
Amphidiploid	none	5.0	82	94	16
G. hirs. 36M2×	*G. hirs.* 1306×*G. barb.* v. *marit.*	7.0	15	100	1
×*G. barb.* 35-1	*G. hirs.* 8517 and 13638	10.0	40	100	4
	G. hirs. 36M2	21.4	14	100	4
G. barb. 35-1×	none	0.0	hundreds	0	hundreds
×*G. Thurberia* Tod.	*G. barb.* 35-2	2.0	42	100	2

The fertility of the anthers, as may be seen from Table 4, increased owing to grafting. There was a clear connexion between this increase and the genetic relation between scion and stock, *viz.* the closer the relation between the stock and scion, the stronger was its positive effect. Thus, the strongest effect was produced upon the triploid and amphidiploid by their parental forms.

The fertility of the ovules was also increased by grafting.

Variation in the number of mature bolls and seeds as dependent on grafting is shown in Table 5 which contains data on the per cent of mature bolls (to the total of flowers) and on the average number of seeds per boll.

Table 5

Scion	Stock	Bolls	Seeds	
			normal	empty
Triploid	none	4	0.25	4.0
G. hirs. 0100×*G. barb.* 35-4	*G. barb.* 35-2	18	0.5	0.7
Amphidiploid	none	4	0.0	1.4
G. hirs. 36M2×*G. barb.* 35-4	*G. hirs.* 13638	6	0.0	3.0

It may be seen from Table 5 that the effect of grafting was to increase the number of mature bolls, as well as (in most cases) the average number of seeds per boll. It should be noted that in this case was observed an after-effect of grafting upon maturing bolls rather than the action itself, since the foliar area of the stock was negligible at the time of the maturing of the bolls as compared with that of the scion.

Grafting was thus found to alter the time of the beginning of blooming in cotton, the intensity of blooming, the intensity of shedding ovaries, the degree of fertility of anthers and ovules and the number of mature bolls.

These alterations depend on the degree of the genetic affinity between stock and scion. The acceleration of blooming, the increase in blooming intensity and in the proportion of maturing bolls were the greater, the more genetically remote was the stock. On the other hand, the increase in the fertility of the anthers and ovules in the scion was the greater, the closer the affinity of the stock.

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PLANT PHYSIOLOGY

ON THE PROBLEM HOW TO AVOID POTATO DETERIORATION IN THE SOUTH

By G. KH. MOLOTKOVSKY

(Communicated by D. N. Prianishnikov, Member of the Academy, 31. V. 1944)

To the main factors responsible for potato deterioration in the south should be referred high temperature and air dryness. T. D. Lyssenko has suggested the method of summer planting ([2,3])* as a means to control deterioration of this crop in the south. When planted in summer, the formation of tubers is postponed to a later, cooler season, and deterioration is thus prevented.

On the basis of the theory of stage development, T. D. Lyssenko is interpreting the phenomenon of deterioration of potato-tubers as a symptom of ageing of the tissue, and, consequently, as an irreversible process ([4]). It has been demonstrated, however ([5, 6]), that if grown under favourable conditions, for example, at northern latitudes, tubers of deteriorated potato can be rejuvenated, which should be interpreted to mean that the ageing process is a reversible one.

If summer deterioration (ageing) be interpreted as a result of accumulation in the tubers of accessory products of metabolism, such as ammonia, amides, ethyl alcohol and other compounds revealed in tubers of deteriorated potato by Rakitin and Shumova ([7]), then delayed tuber-formation in potato plants normally vegetating since spring, and the postponement of this process by means of rejuvenation of the aboveground part of the plant to a later season (say, to August or September), would be a procedure enabling the potato-grower to obtain high-quality, non deteriorated seed material.

On this assumption we removed the part of the foliage of the potato plants at two different dates, so as to make the plants produce rejuvenated leaf organs at the time of the most intense activity of the external agents (June—July).

The leaves of the medium internodes are known to be the most actively eliminating form-building substances ([8]). The same has been found to hold true for potato ([9]).

Our experiment was carried on according to the following scheme: 1) Control, no foliage removal. 2) Control, summer planting. 3) Two apical leaves only cut off. 4) One third of aboveground part removed. 5) A half of the aboveground part removed.

The experiment embraced two potato-varieties, Early Rose and Smyslovsky. Mowing was done at two dates: on June 11th, and on June 21st. At the former date, the foliage of Early Rose attained a mean height of 35—45 cm, that of Smyslovsky, 30—40 cm. On June 21st, the foliage of Early Rose was 44—45 cm high, in Smyslovsky it was 50—60 cm. The formation of individual buds was recorded in Early Rose on June 11th.

The soil of the experimental plot was a dark serozem of piedmont regions, artificially watered. During the growing season watering was applied eight times on the

* Summer planting of potato has long since been practised in the south of France and especially in Algeria (see Gasparin ([1])).

11th, 21st and 29th of June; 7th, 9th and 27th of July; 5th and 28th of August. The experiment was made on plots 60 m² large, at double replication. Area of feeding, 40 × 70 cm. Some of the data obtained are summarized in Table 1.

Table 1

Time of Tuber Formation and Yield as Affected by Removal of Foliage

Experimental set	Start of the tuber formation	Average number of tubers per hill	Average weight of tubers per hill, in g
Variety Early Rose, planted on May 10th			
Control (foliage not removed)	28.VI	20	447
1/3 of foliage removed, on June 11st	8—10.VII	22	416
Same, on June 21st	14—15.VII	13	305
1/2 of foliage removed, on June 11th	24—25.VII	12	398
Same, on June 21st	18—20.VIII	7	198
Variety Smyslovsky, planted on May 10th			
Control (foliage not removed)	10—12.VII	16	372
1/2 of foliage removed, on June 11th	22—25.VIII	20	322
Same, on June 21st	25—27.VIII	9	316
Summer planting done on June 18th (no foliage removal)			
Variety Early Rose	30.VIII—1.IX	15	301
Variety Smyslovsky	5—8.IX	14	298

According to the procedure and date of foliage removal, tuber formation was shifted to later dates, in separate cases delayed by 50 days. Decapitation failed to produce any appreciable results. The most pronounced shifts were obtained in the case of removal of a half of the foliage done at the second date (on June 21st); this may easily be understood, for in this way mature, most active leaves were removed, which are located at the medium internode. The removal of one third of the foliage was not without effect either.

The foliage of those plants which were subjected to the operation of cutting, especially in the sets were half of the foliage was removed, sharply differed from that of the controls. In the first line it was dark green, whereas the leaves of the control were light green. The size of leaf blades, the total amount of green substance and even the shape of the bushes were quite different in the two cases. The experimental plants, in particular those from which half of their foliage was removed, remained green until the very harvest, whereas the foliage of the controls dried off towards the end of August.

It is noteworthy that in the sets involving the removal of a half of the foliage, the formation of young stallons was observed in either variety as late as on the 1st of October. This should be taken to mean that in these experimental sets the rejuvenation of the potato plants was so intense as to shift tuber formation by a rather considerable period of time, owing to which the yield was rather reduced quantitatively. However, the tubers of the plants in these experimental sets were found favourably to compare with those previously described, both in size and vigorous appearance. There was a rather close resemblance between them and the tubers of such experimental plants as had been deprived of a third of their foliage.

The experiments here described permit of the following preliminary conclusion. The removal of a half of the foliage practised at the period of budding results in a rejuvenation of the plant, owing to which tuber formation is made to proceed at a cooler autumn season; in this way non-deteriorated, sound seed material is obtained.

It may be supposed that by duly selecting the dates of foliage removal, it will be possible to obtain yields that will be not inferior to those produced by plants with their foliage intact. But even if the yields will in the subsequent be found to be somewhat reduced by the procedure (which is not likely, according to the experiment described below), this goes in no way to disparage the method whose main value consists in securing non-deteriorated seed potatoes. These tubers, if planted in the next spring, will give a yield much exceeding that obtainable with tubers obtained from routine planting.

K. B. Babaev, Director of the Kazakh Institute for Agriculture, in Alma-Ata, who kindly offered us his experiment field for the experiments described above, has carried out similar experiments at the Talgar experiment station of the Institute. According to the results obtained by him, the removal of foliage results in a shift of the process of tuber formation to a later season, more favourable to the normal development of potato plants.

What is interesting about this latter experiment, is that the potato plants deprived of a half of their foliage mostly gave an increased yield of tubers (Table 2).

Table 2

Yield of Potato as Affected by Foliage Removal
(according to data of the Talgar experiment station)

Variety	Planted on	Foliage removed on	Yield of tubers in kgs		
			no foliage removed	½ of foliage removed	up to 15 cm of foliage removed
Smyslovsky	25.V	5.VII	11.2	17.1	10.6
	20.VI	20.VII	12.4	12.1	18.1
Early Rose	25.V	5.VII	11.2	14.3	9.2
	20.VI	20.VII	12.3	15.6	8.0
Epicurus	25.V	5.VII	12.1	11.6	7.8
	20.VI	20.VII	11.4	18.1	12.7

Especially conspicuous is the phenomenon in the case of the variety Smyslovsky planted late in spring (25.V), in the medium-early variety Early Rose (both in the case of late-spring planting and summer planting) and, lastly, in the early variety Epicurus planted in summer (20.VI).

The results of the experiments here described make us infer that when having recourse to foliage removal, no sound potato tubers can be obtained unless the biological characters of the varieties are duly accounted for. The dates of foliage removal should be chosen so as to obtain normal, or even increased tuber yields.

In case the foliage is cut off at a height of 15 cm above the ground, the period of tuber formation is referred to a later season, while the yield is somewhat reduced as a rule.

In concluding, it can be assumed that the removal of foliage in potato is to be recommended in the Southern Kazakhstan (and possibly in other southerly regions of this Country, as well) as a means to avoid deterioration of potato.

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PHYSIOLOGY

CONDITIONED-REFLECTORY DIFFERENTIATION OF CALLS IN *PASSERES* AND ITS BIOLOGICAL VALUE

By A. N. PROMPTOV and E. V. LUKINA

(Communicated by L. A. Orbeli, Member of the Academy, 29. III. 1944)

1. The high degree of perfection of the vocal apparatus in birds is closely connected with their biology; it has undoubtedly been attained as a result of a prolonged work of natural selection (3). The power of birds to fly, which enables them of rapid locomotion, makes the vocal communication between separate individuals of paramount importance (loud specific calls). This is especially true of silvan birds which dwell in foliage and wood-thicket. Nocturnal migrations (12), accumulation into flights, the relations between the two sexes in reproduction (7), and many other features peculiar to the biology of birds are also intimately connected with the high degree of perfection of their vocal apparatus.

2. Every bird species is characterized by calls, or specific shrieks (10) uttered under definite biological conditions. In certain species (especially in *Turdiformes*, *Paridae* and some others) these calls have attained a rather great diversity, offering considerable interest from both biological and physiological point of view. Physiologically many of these calls are a purely reflectory external expression of a certain affect, *i. e.* are undoubtedly due to humoral stimuli. Sometimes they are also kinaesthetically correlated to some definite movements of the bird; such are, for example, the calls which accompany the arched flight in *Motacillidae*, «bows» and springs in many *Turdiformes*, *etc.*

Particular attention should be attached to the calls of birds, viewed under their biological aspect. In what follows are summed up the results of special observations and experiments carried on during six years both in nature and under laboratory conditions. Our special study of the biological value and physiological nature of calls was carried out mainly on *Parus major* L., a species in which the calls are highly diversified.

Along with this species, however, the specific value of calls was also investigated in various birds, and what is reported below refers in principle to other *Passeres*.

3. Aside from the song which in *P. major* is subject to considerable variation both in pitch-standard and rhythm (10), up to twenty different calls or shrieks have been recorded for this species. One who was merely to observe those birds under natural conditions would not fail to become aware of the fact that each of their calls has a different biological meaning, *i. e.* is uttered by the birds under definite circumstances only. Thus, for example, the calls they exchange when at a large distance from one another differ strongly from those uttered when at a close distance; quite peculiar a shriek is uttered at the approach of a raptorial bird. Anxiety, alarm, warning, threat, finding of food, relations between male and female, especially at the period of nesting, surprise at an unexpected encounter, or even when some rather

inoffensive stimulus is suddenly applied,—all these are accompanied by special calls.

The sounds can scarcely be expressed either in notes of a definite key, or by letters ([10], [9]); but they are absolutely constant, though assuming a certain peculiar intonation in different individuals. By applying definite artificial stimuli, optical or acoustic (say, by showing a raptorial bird, alluring or imitating various calls) it is possible to provoke in wild birds regular typical response-reactions, *i. e.* to produce in them a definite physiological state; such tests have been carried out in very large numbers. Such observations, especially if carried out in autumn and winter when the *Paridae* live in flights, bear out the following circumstance which is of great biological significance. Any call uttered by a single individual in response to a definite stimulus, serves as a specific signal to all those individuals which can hear it. Thus, the state of the individual that uttered a call, say, of fright, is at once transmitted to its neighbours. It is sufficient, for example, to make a single individual in a flock of *Paridae* to utter a definite call, to observe a specific reaction in the rest of the birds.

4. Such an adequate and highly differentiated response to definite sound stimuli makes one assume the existence of conditioned reflex connexions that are formed individually, *i. e.* by way of experience. However, this assumption is at the first sight somewhat compromised by the fact that any call connected with some violent affect (say, alarm, fright, fighting) is found to be inherent and appearing almost typically without any training ([1–6], [15], [16]). Experiments carried out with young birds that had been brought up without parents and had never heard specific calls of alert, showed ([11]) such calls to provoke in them at once a pronounced specific response, *i. e.* to act as an unconditioned stimulus. It is noteworthy that a call uttered in the case of violent fright has not only the value of a specific stimulus, but also provokes a reaction of fright in many other species of *Passeres*, though it is rather specific as to its vocal characteristic. Unconditioned are also such calls as are uttered by the birds in the case of definite movements (see above).

In this case we have undoubtedly to do with profound kinaesthetic correlations between the general musculature of the body and the vocal muscles.

We thus see that there can be no doubt as to the unconditioned nature of such calls as are connected with violent affects. But assiduous observations on birds under natural conditions, and especially experiments on young *Paridae* brought up under laboratory conditions went clearly to support the correctness of the assumption that there is also differentiation of calls due to conditioned reflexes. What is important here, is the biological differentiation. It has been found that though showing a great variety of calls, young *Paridae* when brought up without parents, fail to develop such calls as are specifically connected with definite situations and are typical of the species, save for shrieks accompanying fright, fighting or calls at a large distance; neither do they respond to the calls of wild individuals as young birds do under natural conditions. If we are to compare the vocal repertory of young birds brought up in the laboratory with that of wild individuals, we become at once aware of the disorderly character of the calls in the former, while in the latter the calls corresponding to the various biological situations display a well-pronounced specific differentiation. It is evident that such a specific differentiation is attained, perfectioned and stabilized under natural conditions owing to innumerable stereotypic occurrences of such situations as require no less stereotypic motor, or any other, reactions ([9], [11]). In the «herald» a definite signal (call) may be absolutely involuntary (unconditioned), but the response it provokes in the «auditory» will be no merely unconditioned «response», as, for example, in the case of shriek occasioned by fright, but also a specific conditioned-reflectory «superstructure» which has determined their subsequent behaviour. It may be supposed that an important part in the formation of such «superstructures of behaviour» is played by the principle of «actor and spectator».

5. The biological value of the differentiation of calls may scarcely be overestimated. It is the chief means of intercourse between separate individuals of the spe-

cies, male and female (7), parents and young (13). Such a differentiation permits of a high degree of correlation of all the members of a flock, contributing greatly to their collective defense against rapacious beasts and other perils (14). The existence of adequate conditioned reflectory connexions between a definite sound stimulus (call) and the motor reaction permits of a rapid conveyance of a given physiological state from one individual to many others which in their turn «broadcast» it to their neighbours (individual and specific «heralds» in mixed flocks). This remarkable signal-system of stimuli and responses, which is based on a most subtle work of the acoustic analyser and on most complex kinaesthetic correlations, plays a most important part in guaranteeing the viability of both individual and species under natural conditions.

A further treatment of this problem will therefore be not only of purely physiological, but of evolutional interest, as well.

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